

# Data Collecting and Processing for Quasi-Geoid Determination in Brazil

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**Abstract.** As a country of continental dimensions, with some of the world's largest gaps in terms of gravity data coverage, Brazil has been working on gravity surveys to reach its decimeter geoid. Most of the existing gravity surveys were carried out in areas with oil exploration potential and, therefore, are not evenly distributed. Taking this into account, some organizations have been concentrating their efforts to achieving a more homogeneous gravity data distribution to minimize this restriction. Besides these efforts, Digital Terrain Model (DTM) has been considerably developed by the digitization of many topographic maps during the last few years. Geopotential model EGM96 (GSFC/NIMA-NASA) has also been extensively compared with known gravity anomalies and GPS levelling geoid heights. Based on these improvements, the results of the computation for a Brazilian quasi-geoid with 10' resolution are presented.

**Keywords.** Gravity, Geoid

## 1 Introduction

The creation of SCGSA in 1993 (Sub-Commission for the Geoid in South America- now Sub-Commission for Gravity and Geoid), provided the possibility of a better integration of South American countries in determining a more accurate geoid for each of them and for the continent itself. For this purpose, all existing gravity and GPS data over benchmarks have been made available in order to form a continental data base for geoid computations. New agreements have also been made and the inconsistencies between countries' gravity and altimetric networks, as soon as detected, are being analyzed. Some gravity surveys lack documentation regarding their coordinates' reference system and gravity value origin. So,

validation processes are essential to identify the reliable data and the priority areas for new surveys, both for filling gaps and for crosschecking the existing data. With the efforts of SCGGSA, new gravity points in different countries were validated and added to this new computation. The continuous digitization of new topographic maps in Brazil, Argentina and Chile allowed some improvements in the DTM.

## 2 Gravity and GPS-Levelling Data

South American Gravity Project [Green & Fairhead, 1991] was the first great effort in collecting and validating gravity data in this continent. This initiative was important to indicate the terrestrial and marine gravity points distribution and the identification of the major gaps. In 1991, the Anglo-Brazilian Gravity Project (ABGP), a cooperation program between EPUSP, IBGE and GETECH, supported by NIMA, started some new efforts to fill the gaps in Brazil. After seven years of activities, this project was responsible for an outstanding improvement on the gravity point distribution, mainly in the Amazon region, including rivers and airstrips throughout small villages in Amazonas state. For this, IBGE expanded the Basic Gravity Network established by National Observatory as part of a strategy for future reoccupations and links with other South American countries that border Brazil.

The activities of ABGP were extended to other countries in the continent as South America Gravity Studies (SAGS). Due to these projects (ABGP and SAGS), a set of about 30,000 new gravity points in the last 10 years could be added to improve the quasi-geoid computation. Many important areas remain to be surveyed but efforts are being envisaged to do airborne gravity where traditional terrestrial surveys are extremely

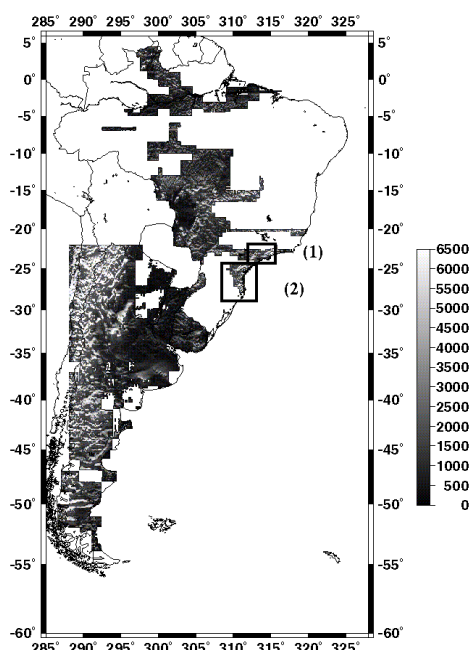
difficult to execute, as in Amazon forest. IBGE has also systematically been performing gravity determination over every new spirit levelling survey and, when possible, over the old existing ones.

A basic 5'x5' file (from 25° N to 60° S to 100° W to 25° W) was formed with mean free-air and Bouguer anomalies values in land, satellite altimetry free-air anomaly in marine areas, KMS model [Andersen & Knudsen,1998], mean height (depth offshore) and mean terrain correction. The file is kept updated with all reliable data made available. A 5' resolution is far from the possibility to be reached at the moment. The mentioned basic file of 5' has been used to derive 10' mean anomaly values which is less uncomplete.

Several countries carried out GPS observations on bench marks of the geometric levelling. A total of 418 points are now available.

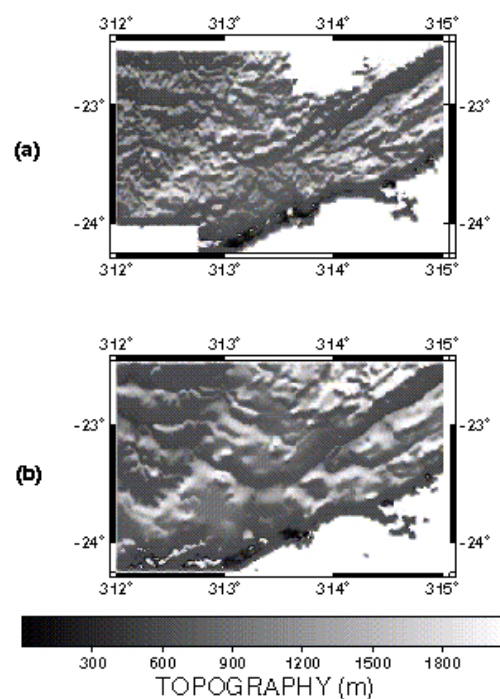
### 3 DTM

Considering the importance of a Digital Terrain Model (DTM), an effort has been addressed in the last few years for digitizing topographic maps in Brazil, Argentina and Chile (Figure 1).

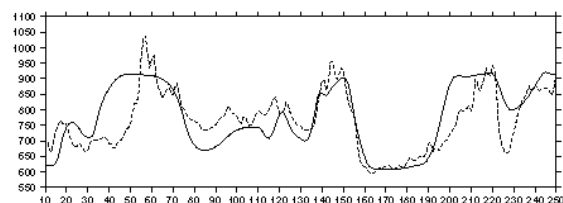


**Fig.1.** Maps digitized and areas (1) and (2) for comparisons between EPUSP and GLOBE models

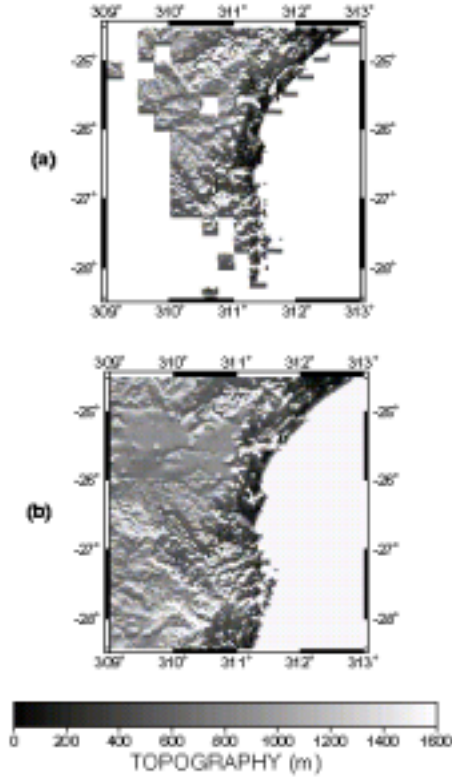
Many areas still exist, in particular in Brazil, where the work is still not completed and, in these cases, the gaps were filled with the GLOBE model [Hastings et al, 1999]. Two areas, (1) and (2), were selected in figure 1 for estimation of the agreement between the so called EPUSP model and the GLOBE (figures 2 and 4). The profiles in figures 3 and 5 show comparisons of these models. Both models show no apparent shift in the reference coordinates. Nevertheless, the EPUSP model presents a better resolution and can represent better the short wavelengths of the topography.



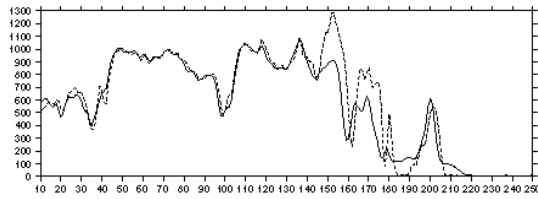
**Fig. 2.** Area (1) – EPUSP (a) and GLOBE (b) models.



**Fig. 3.** Area (1) - Differences between EPUSP (dotted line) and GLOBE (solid line), (heights in Y axis) ( $\lambda=312.4^\circ$ ,  $\varphi=-24^\circ$  to  $\lambda=315^\circ$ ,  $\varphi=-23^\circ$ )



**Fig.4.** Area (2) – EPUSP (a) and GLOBE (b) models.



**Fig. 5.** Area (2) - Differences between EPUSP (dotted line) and GLOBE (solid line), (heights in Y axis) ( $\lambda=310^\circ$ ,  $\varphi=-27^\circ$  to  $\lambda=312^\circ$ ,  $\varphi=-25^\circ$ )

#### 4 Mathematical Formulation

The computation has been carried out using basically the modified Stokes's integral approach. The well known formulas have been used:

$$\delta N_l(\theta, \lambda) = \frac{R}{4\pi\gamma} \int_{\psi=0}^{\psi_o} \int_{\alpha=0}^{360} \Delta g(\alpha, \psi) \delta S_l^m(\psi) \sin \psi d\psi d\alpha \quad (1)$$

with  $\delta S_l^m$  the modified Stokes kernel derived from a convenient modification [VANICEK et al., 1987]. Without derivation the formulas are:

$$\delta S_l^m(\psi) = \delta S_l(\psi) - \overline{\delta S_l}(\psi) \quad (2)$$

with

$$\delta S_l(\psi) = S(\psi) - S_1(\psi) \quad (3)$$

$$S_1(\psi) = \sum_{n=2}^l \frac{2n+1}{n-1} P_n(\psi) \quad (4)$$

$$\overline{\delta S_l}(\psi) = \sum_{i=0}^l \frac{2i+1}{2} t_i P_i(\cos \psi) \quad (5)$$

The gravity anomaly used in (1) for this paper was the Helmert gravity anomaly [Martinec et al., 1993], with long wavelength component removed using EGM96 geopotential model up to degree and order 50. As a consequence a limit of integration of  $3.6^\circ$  has been adopted.

The FFT (Fast Fourier Transform) technique has been extensively used up to now due to the well known computation facilities. The theory and the equations related to this formulation in planar approximation can be found in [Schwarz et al., 1990] and [Blitzkow, 1999].

Instead of the planar approximation, the spherical approximation to the Stokes's integral can be used. The integral can be carried out in two (2D FFT) or one (1D FFT) dimension [Strang Van Hees, 1990]. The option of 1D FFT has the advantage that the kernel can be exactly defined without any approximation [Haagmans et al., 1993].

Very recently, W. Feartherstone introduced a modified kernel called spheroidal Molodenskii – Meissl [Feartherstone, 2000]. For this paper the computation has been done using numerical integration according to formulas (1) to (5) in a  $10'$  grid from  $10'$  mean anomalies. Experiments are in process with different FFT software.

#### 5 Results

For the evaluation of the results, the quasi-geoid values were compared against GPS geoidal heights derived from GPS observations on bench marks (in Brazil and other South American countries) in a total of 418 points. Figure 6 shows a quasi-geoid model for Brazil computed from the present data available.

The derived GPS geoid heights have been compared with the EGM96 geopotential model.

Two different strategies had been selected: degree and order 180 and 360, referred to WGS-84. The zero order terms (mean difference values) and the RMS differences are presented at the Table 1. Finally, a comparison has been made with gravity derived quasi-geoid (grav-geoid) height computed in the same GPS points using 10' mean gravity anomalies.

**Table 1.** Comparisons between GPS geoid heights, EGM96 and grav-geoid.

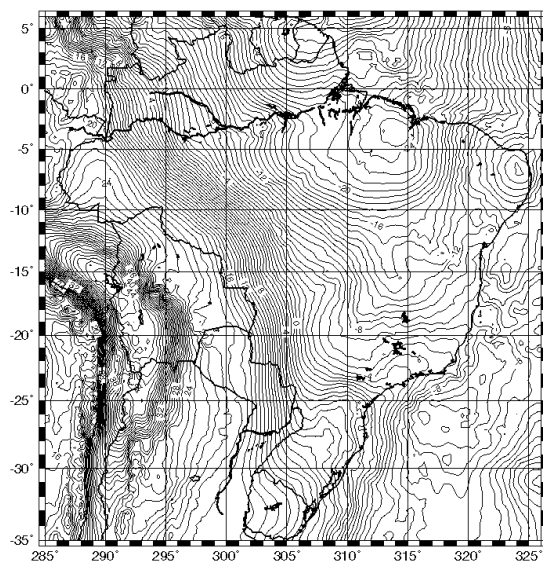
Model	N <sub>0</sub> (m)	RMS diff. (m)	Degree/ order
EGM96 (WGS-84)	-0.28	0.98	360
EGM96 (WGS84)	-0.29	1.00	180
Grav-geoid (WGS-84)	-0.30	1.26	-

## 6 - Conclusions

The RMS differences at the Table 1 show that EGM96 up to degree and order 360 fits slightly better to the GPS heights than up to 180. The grav-geoid, on the other hand, has an RMS greater than EGM96. This fact has to be investigated.

The GLOBE model can certainly improve the DTM in areas where topographic maps have been digitised. The new model derived at EPUSP will be used to improve the future geoid computations.

Data validation is a continuous and laborious process and corresponds to the main priority of EPUSP and IBGE in terms of gravity and geoid.



**Fig. 6** Quasi geoid model for Brazil.

## 7 Acknowledgement

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