# Optimal Integration of SIRGAS2000 in ITRF2000 using Minimum Constraints Approach

## Àrea de interès : Geodesia

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## Abstract.

Since 1988, with the establishment of IERS (International Earth Rotation Service), efforts were undertaken to realize precise reference frames through spatial positioning techniques, like VLBI (Very Long Baseline Interferometry), SLR (Satellite Laser Ranging), GPS (Global Positioning System) and DORIS (Doppler Orbitography and Radio-positioning Integrated by Satellite). With the contribution of solutions by Global Analysis Centers, a series of reference frames called ITRF (International Terrestrial Reference Frame) were realized and improved, version-by-version, until the latest version called ITRF2000. The densification of this global frame by regional frames like EUREF (European Reference Frame), NAREF (North American Reference Frame), and SIRGAS (Geocentric Reference System for the Americas), was the solution for many countries to integrate their geodetic networks into a precise global frame.

The SIRGAS project is in the second realization of its reference frame. The first was established in May 1995, comprising 58 stations in South America. The second network observation was in May 2000, comprising 182 stations in the three Americas and the Caribbean islands. The goal of this paper is to present the best strategies for the integration of an ITRF regional densification, using SIRGAS2000 network as an example. SIRGAS2000 daily solutions were generated with the Bernese software, version 4.2, with removable constraints. The combination into a final solution was achieved using the minimum constraints approach, preserving the original characteristics of the daily solutions and providing the assessment to ITRF2000 reference frame. In this proposal, eleven IGS stations in the ITRF2000 were used do define the SIRGAS2000 reference frame. The criteria for station selection were the same as used by ITRF. The software used to perform this task is CATREF, developed by LAREG/ENSG. Besides coordinates, and their standard deviations, results of a 7-parameter transformation are presented and analyzed using different strategies of combination. In this work, velocities will not be considered, because only 10 days of solutions were combined.

**Keywords.** Reference systems, Regional Network, Free net adjustment

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## 1 Introduction

There are several strategies and mathematical models developed for the integration and combination of reference frames. Generally, the final objective of a geodetic frame is the best datum definition, e.g. origin, scale and orientation.

The subject of this paper is to present the best way of integrating a densification network into an existent reference frame, in other words, expressing a regional solution in ITRF. For this proposal, two reference frames are used as examples, one with global (ITRF2000) and the other (SIRGAS2000) with regional characteristics.

Basically, two strategies are commonly used by analysis centers to generate combined solutions. One is a constrained solution and the other is an unconstrained, also called "free network" solution. A constrained solution is achieved by tying ITRF stations with heavy weights into the network. This solution is successful if a priori and new coordinates are in a linear domain, otherwise the network will be distorted. The unconstrained solution, implementing a minimum constraints approach, preserves the original characteristics of the network and at the same time provides the alignment to ITRF (7-parameter transformation). Here, we will concentrate on the procedure for the combination and implementation strategy of SIRGAS2000 using an unconstrained solution.

The modeling of a combined unconstrained solution is fully explored in Altamimi[2002], and will not be presented here. We will begin by presenting information regarding ITRF2000 and SIRGAS2000 frames. Some strategies of combination are explored (unweighted and weighted least square adjustment) and results are presented indicating the best solution. The last step is the selection of the best ITRF2000 stations with good quality and agreement with SIRGAS2000, in order to define SIRGAS reference frame. To conclude, results in terms of datum definition as well as the quality of the SIRGAS realization will be presented.

## 2 ITRF2000 and SIRGAS2000

Since 1988, with the creation of the IERS (International Earth Rotation Service), a series of realizations of a Global Terrestrial Reference System – ITRS were carried out and continually improved. The series of ITRF represents the system in practical sense, using high precision geodetic observations by permanent geodetic stations. Since ITRF94, 5 geodetic techniques are used for this purpose, i.e. VLBI, LLR, SLR, GPS and DORIS. In April of 2001, the latest release of the global reference frame, called ITRF2000, was completed. ITRF2000 contains about 800 stations located at about 500 sites. For the first time it was a solution free of a tectonic plate motion model [Altamimi, et al. 2002]. Sites of high geodetic quality established in ITRF97, were used with minimum constraints, ensuring a datum definition at the mm level. For this purpose the ITRF2000 datum definition is given by:

(1) Scale and rates: weighted mean of 5 SLR and 3 VLBI solutions;
(2) Translations and rates: weighted mean of 5 SLR solutions;
(3) Orientation: is aligned to that of ITRF97 at epoch 1997.0;
(4) Orientation rate: No-net-rotation rate with respect to NNR-NUVEL-1A, through the selection of ITRF sites with high geodetic quality.

The SIRGAS2000 reference frame was established in a period of 10 days of GPS observations, from May 10$^{th}$ to 19$^{th}$, 2000, with about 180 stations covering the three Americas. It is considered a regional densification of ITRF2000 on the American continent [Luz et al.,1999].

The data in each station was organized in 24 hour periods and converted to RINEX format using 30 seconds sampling rate. In a second step, some information regarding station and antenna/receiver identifications, according to IGS standards, as well as antenna heights, were verified. For the processing combined IGS ephemeris together with consistent ERP (Earth Rotation Parameters), and Global Ionosphere Maps (GIMs), with the goal to reduce the ionosphere refraction, were used. The software used was Bernese GPS Software, version 4.2, and the options adopted for daily processing were:

– During data processing, the double phase differences were modeled in L3, using an elevation mask of 10° and a sampling rate of 30 seconds;

- The strategy forming the single difference phase files was OBS-MAX[1];
- The ambiguities were solved using QIF (Quasi Ionosphere Free) strategy, in baseline mode;
- No *a priori* troposphere model was used;
- The troposphere parameters were estimated in all steps of parameter estimation. The corrections of the troposphere delay at zenith for each station were estimated every 2 hours, having 12 daily correction numbers. Neill mapping function was adopted to compute corrections;
- The elevation-dependent weighting is applied using the function cos(z)**2.

Considering the great number of stations, SIRGAS2000 network was divided in blocks for processing. The criteria adopted to divide the whole network in blocks was by latitude zones and the quantity of blocks was chosen in terms of processing time and memory space supported by the computers. Following these requirements it was necessary to split set up 9 blocks with approximately 24 stations each. Each block comprises about 6 ITRF stations providing the link between them. In the last run of processing normal equations were constrained, with sigma value of 2 mm. For CATREF input it was necessary to convert normal equations given in Bernese format to SINEX format, which was done using the ADDNEQ[2] program. For each daily solution ADDNEQ was executed twice, the first run to produce a free network solution, the second run to generate SINEX format with removable constraints using the coordinates produced in the first step as a priori values. Considering the 10 days of observations in 9 blocks, 90 daily solutions in SINEX format were thus obtained.

## 3 Combined solution using CATREF software

The CATREF Software was used to combine the daily/block solutions, using the option of minimum constraints approach implemented in that software, as presented in Altamimi [2002].

The steps to carry out a free network solution, implementing the minimum constraint strategy is:

(1) Remove constraints from daily SINEX files;
(2) Apply (or add) minimum constraints, only to define a TRF-origin, scale and orientation;
(3) Create SINEX file with ITRF2000 stations and coordinates propagated to SIRGAS epoch 2000:135;
(4) Combine daily or block solutions: Each resulting combined solution is expressed in ITRF2000 at the central epoch of the daily/block solution;
(5) Variance factors are estimated for all individual solutions included in each combination;
(6) Check internal consistency of combination in each block, looking for residual outliers (screening criteria : normalized position residuals are bigger than 4, >1 cm);
(7) Combination is repeated and individual solutions are re-scaled until no outlier is found in solutions. This iterative process is to refine the estimation of variance factor. Table 1 shows these values estimated by block combination;
(8) Compare daily/block combined solutions with ITRF2000 in order to see network effects through transformation parameters;
(9) Eliminate ITRF2000 stations from SINEX propagated file with residuals higher than 2 cm and those with only one solution in ITRF2000;

Table 1: variance factor estimated in each block of SIRGAS network.

| Block identification | $\sigma_0^2$ | Latitude limits |
|---|---|---|
| SIRGASN | 4.474 | 61° N to 45° N |
| SIRGASN1 | 4.396 | 45° N to 25° N |
| SIRGASN2 | 4.190 | 25° N to 10° N |
| SIRGASC | 5.211 | 10° N to 00° |
| SIRGASS | 5.572 | 10° N to 15° N |

[1] Maximum number of observation between two stations.
[2] Program for combination of solutions in Bernese software.

| | | |
|---|---|---|
| SIRGASS1 | 4.786 | 00° to 20° S |
| SIRGASS2 | 4.909 | 15° S to 35° S |
| SIRGASS3 | 4.933 | 20° S to 35° S |
| SIRGASS4 | 4.391 | 35° S to 62° S |

## 4 Strategies of Combination

Four strategies were carried out in order to choose the final solution. In these tests, the blockwise combined partial solutions were used:

(1) Combining all blocks assuming that they are in the same frame of ITRF2000, imposing that transformation parameters are equal to 0;
(2) Combining all blocks and estimating transformation parameters, meaning that the combined solution is expressed in ITRF2000;
(3) Combining all blocks constrained to ITRF2000, attributing high weights to the 30 ITRF2000 stations. These stations were included in the combination of block solutions;
(4) Combining all blocks applying variance factor computed in solution (3) and the strategy of solution (2).

Table 2: Translation parameters between ITRF2000 and the four test combinations.

| SOLUTION | T1 (cm) / σ ± | T2 (cm) / σ ± | T3 (cm) / σ ± | $\sigma^2$ | EPOCH |
|---|---|---|---|---|---|
| (1) | -.05 /.13 | -.07 /.13 | .14 / .23 | 1.696 | 2000:135 |
| (2) | -.18 /.12 | .06 / .13 | .30 / .22 | 1.537 | 2000:135 |
| (3) | .00 / .12 | .00 / .12 | .00 / .22 | 1.113 | 2000:135 |
| (4) | -.16 /.11 | .04 / .12 | .29 / .19 | 1.338 | 2000:135 |

Table 2 lists translation parameters between ITRF2000 and the four test combinations. In order to assess the magnitude differences compared to ITRF2000, only translation parameters are estimated, given the regional nature of the implied network. From 50 ITRF2000 stations present in the SIRGAS2000 frame, about 30 were used in the tests. Considering that the basic idea is not to develop a constrained solution given by option (3), and that the other three options agree quite well, the solution (2) was chosen to develop the final combination.

## 5 ITRF2000 Station Selection

The best datum definition for SIRGAS2000 frame is achieved by the selection of ITRF2000 sites with good quality. This selection followed the criteria: ITRF stations of high quality, good distribution over regional network, long observation history, located on rigid parts of tectonic plates, more than one solution and technique for estimation, and residual less than 5 mm for coordinates.

Eleven stations( KOUR, AREQ, SANT, LPGS, RIOG, WES2, FLIN, ALBH, CHUR, YELL and STJO) in ITRF2000 were chosen and unfortunately two of them are in deforming zones, because in South America there are few stations that could fulfill all requirements mentioned before. For these stations, ITRF2000 coordinates were propagated to epoch 2000.4, using the relation $X_{2000.4} = X_{1997.0} + \dot{X}_{1997.0}(2000.4 - 1997.0)$ and ITRF2000 velocities.

Figures 1 and 2 show the distribution of all ITRF2000 stations in SIRGAS2000 network and the distribution of the eleven ITRF2000 stations chosen as a-priori information for the final solution. ITRF2000 stations are represented by triangles and SIRGAS2000 by dark dots.

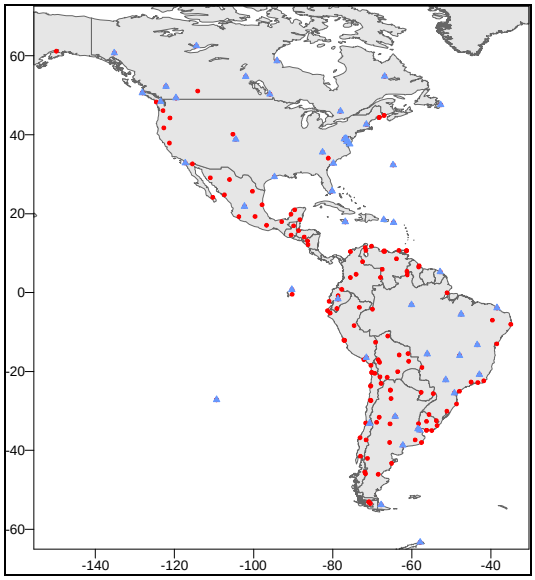


Figure 1

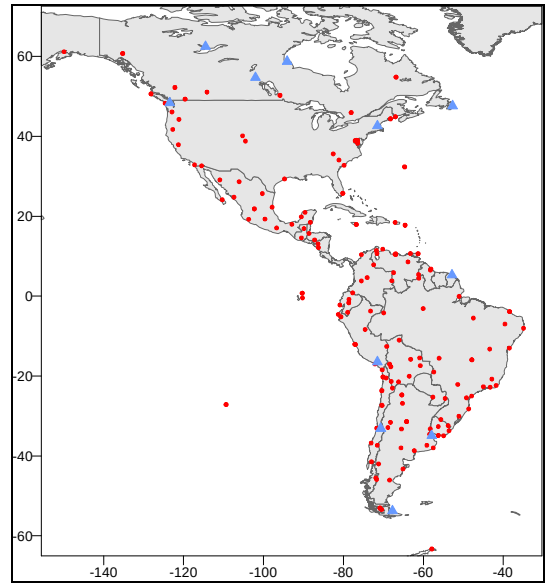


Figure 2

## 6 Evaluating Datum Definition

As mentioned before, a TRS realization through a TRF, requires 7 transformation parameters. They are the relative values between two TRFs. To assess the differences between two frames, ITRF2000 and SIRGAS2000, 7 transformation parameters were estimated. The values of these parameters depend on the weight matrix as shown in Tables 3 and 4, which can assume three different forms (identity, diagonal or full matrix). The Tables show, that values in the 3 tests are equivalent and their differences are smaller than the formal errors. To generate these results, only the 11 ITRF2000 stations mentioned in section 5 were included.

Table 3: Translation parameters between the ITRF2000 and the daily combination.

| | $\sigma^2_0$ | Tx / ± σ | Ty/ ± σ | Tz/ ± σ |
|---|---|---|---|---|
| Identity matrix | .004 | .00/ .15 | .00/ .15 | .00/ .15 |
| Diagonal variance matrix | 1.284 | .05/ .10 | -.07/ .15 | -.18/ .15 |
| Full variance matrix | 2.361 | -.09/ .19 | .10/ .20 | -.01/ .34 |

Table 4: Translation parameters between the ITRF2000 and the block combination.

| | $\sigma^2_0$ | Tx / ± σ | Ty/ ± σ | Tz/ ± σ |
|---|---|---|---|---|
| Identity matrix | .004 | .00/ .16 | .00/ .16 | .00/ .16 |
| Diagonal variance matrix | 1.225 | .11/ .10 | -.15/ .16 | -.17/ .15 |
| Full variance matrix | 2.266 | -.09/ .18 | .09/ .19 | -.02/ .33 |

## 7 Results and Comparison of Different Strategies

The results of SIRGAS2000 reference frame are: coordinates at epoch 2000.4 and their formal errors, transformation parameters (at 2000.4) with respect to ITRF2000, and final combined solutions and residuals.

Following the strategy (2), chosen to generate the final solution, the whole network was combined in two different ways, in order to assess the network effect. One option is the combination of all daily solutions and the other option is the combination of block solutions. The transformation parameters between each result, daily and block, to ITRF2000 (including all ITRF2000 stations) and their rms is shown in Tables 5 and 6, respectively. As can be seen, rotation and scale are meaningless in these results and translations and rms do not have sensible differences, with a slight advantage for the block combination.

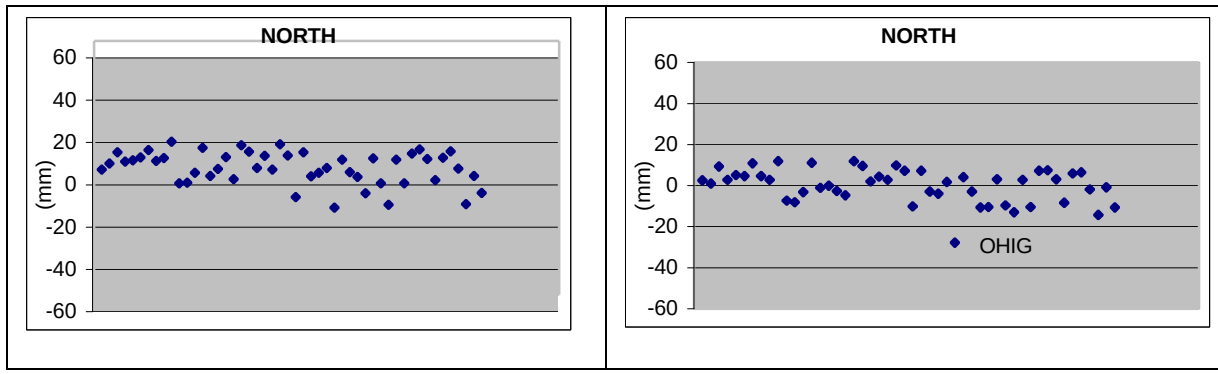
Table 5 : Transformation parameters (unweighted) and rms of transformation between daily combination and ITRF2000.

| Tx/±σ (mm) | Ty/±σ (mm) | Tz/±σ (mm) | Rotx/ ±σ (″) | Roty/ ±σ (″) | Rotz/ ±σ (″) | Scl. /±σ (mm/Km) | Rms (mm) |
|---|---|---|---|---|---|---|---|
| 3.2 / 1.7 | 0.4 / 1.7 | 11.4 / 1.7 | 0.0002 / 0.0002 | -0.0001/ 0.0001 | 0.0000/ 0.0001 | 0.0004/ 0.0004 | 12.3 |

Table 6 : Transformation parameters (unweighted) and rms of transformation between block combination and ITRF2000.

| Tx/± σ (mm) | Ty/±σ (mm) | Tz/±σ (mm) | Rot x/ ±σ (″) | Rot y/ ±σ (″) | Rot z / ±σ (″) | Scl. / ±σ (mm/Km) | Rms (mm) |
|---|---|---|---|---|---|---|---|
| 3.8 / 1.6 | -0.6 / 1.6 | 9.6 / 1.6 | 0.0001 / 0.0002 | 0.0000 / 0.0001 | 0.0000/ 0.0001 | 0.0003/ 0.0004 | 11.6 |

The residuals obtained after Helmert transformation between ITRF2000 and the daily/block combined solutions (which are expressed in ITRF2000, using minimum constraints approach) are shown on Figures 3 and 4, with each component in a local system. The North component presents the highest residual for station OHIG in both solutions, the same happens with SIO3, INEG and EISL in East component and CRO1 in up component. As expected, the Up component has the highest residual values, reaching 5 cm in CRO1, but mean values are 1 cm. Figure 5 shows the residuals obtained after Helmert transformation between ITRF2000 and a constrained solution combined in blocks.



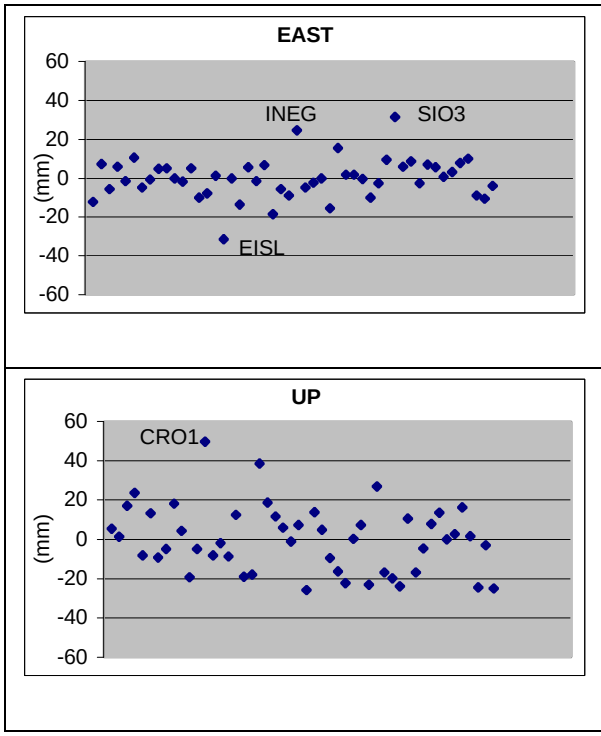



Figure 3: ITRF2000 residuals in daily combination

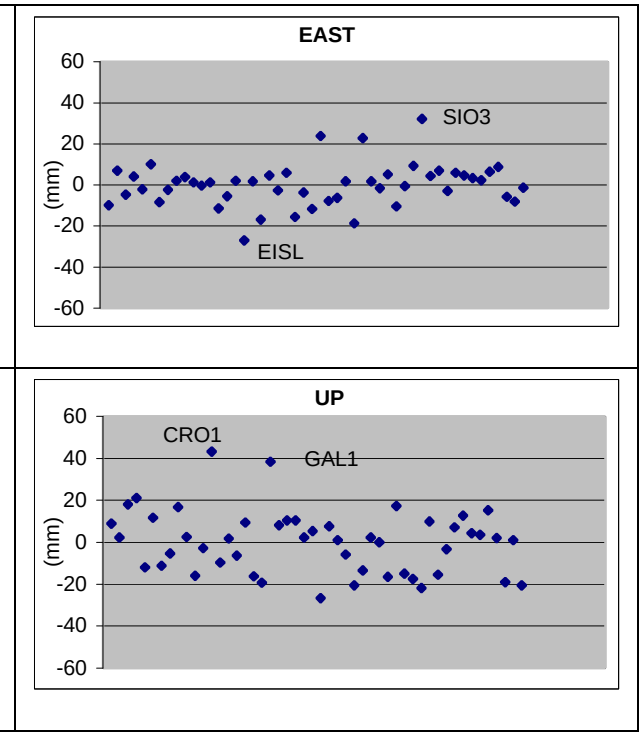
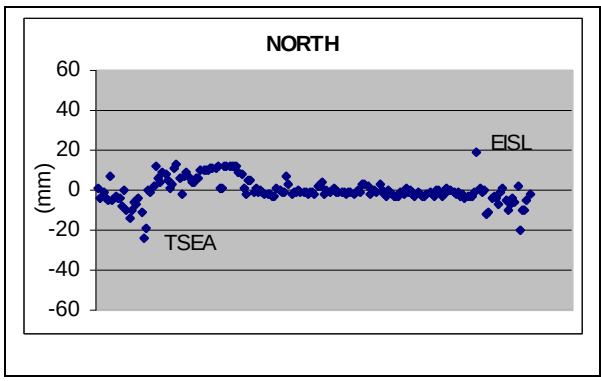
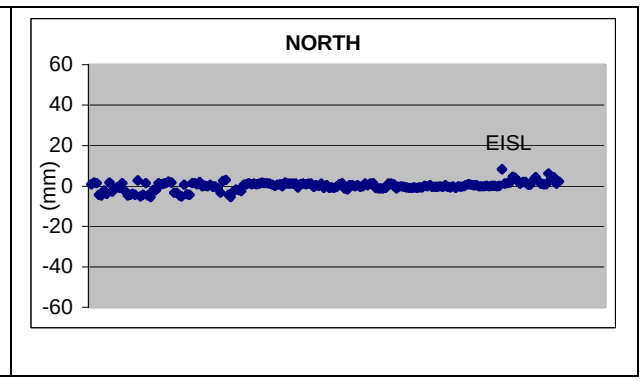



Figure 4: ITRF2000 residuals in block combination

Figure 5 shows the residual differences between constrained and unconstrained solutions. For this comparison, the daily combination was chosen as unconstrained solution. Figure 6 shows the residual differences between the two unconstrained solutions (daily and block). As expected, residual differences in Figure 6 are about 3 times higher than in Figure 6, demonstrating implicitly the deformation caused by the constraints approach, while residual differences between unconstrained solutions are not bigger than 1 cm. Continuing our analysis on Figure 6 it can be seen, that both solutions agree within +/-5 mm in the horizontal components and +/-10 mm in the height components.



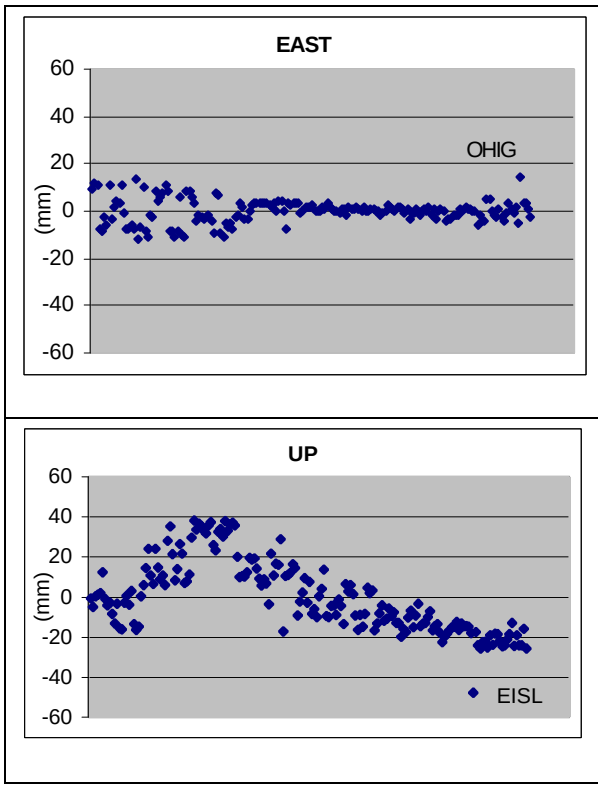



Figure 5: Residuals differences between unconstrained and constrained solutions.

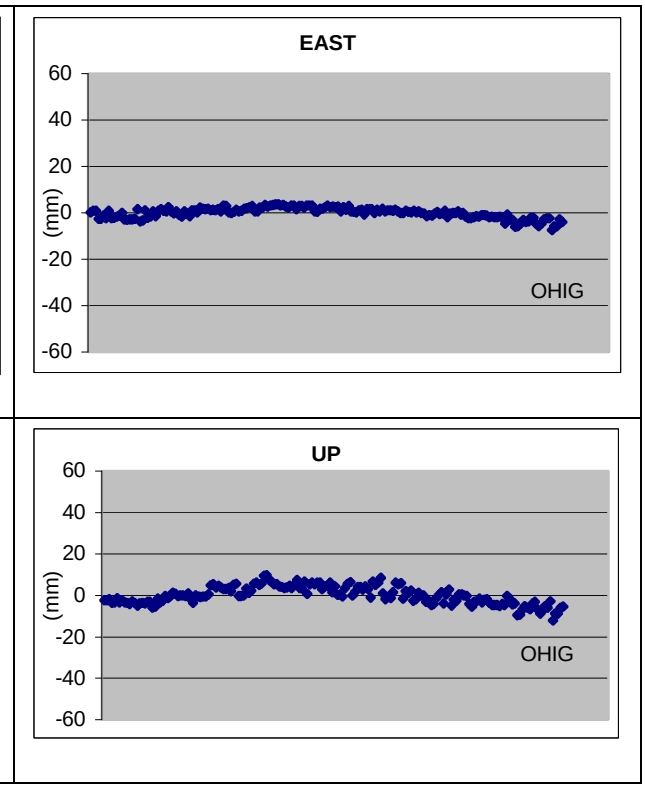



Figure 6: Residuals differences between two unconstrained solutions.

Figure 7 presents a histogram with formal errors, or standard deviations of positions, in an unconstraint solution, showing that about 63% of station positions have an error less than 1 cm, while Figure 8 presents standards deviations of positions in a constraint solution. The errors plotted in Figure 7 indicate that station positions are naturally not all of the same quality, errors in Figure 8 are not realistic since about 90% of station errors range within 5-7 mm, indicating that minimum constraints approach preserve the original solution quality.

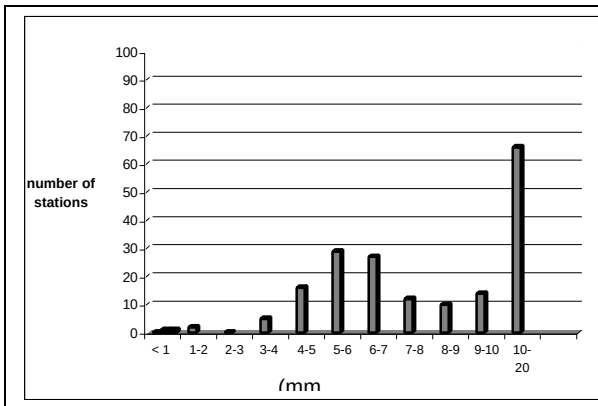



Figure 7: position errors (standard deviation of coordinates) of unconstrained daily combination.

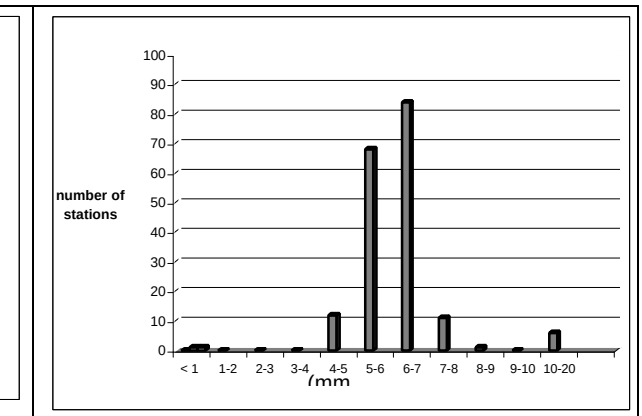



Figure 8: position errors (standard deviation of coordinates) of constrained combination.

## 8 Conclusions

This paper presented two different strategies of network combination and reference frame definition. It can be concluded that there are several strategies to integrate a regional solution in the global ITRF frame, having different impacts on the results influenced by the weighting, network configuration and quality of observations. Analyzing the two adjustment strategies shown in the results section, the free network solution with minimum constraints approach, allows the integration of the SIRGAS2000 network in the ITRF2000, keeping this way its internal and original consistency.

## 9 References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